

Validity Counterexamples, And an Informal Test of Validity

1. Validity Counterexamples.

Understanding logic as the study of **validity** – whether the conclusion of an argument **follows from** its premise(s) – we begin developing a test of validity. In its first form this test will remain informal and intuitive; but it will contain the elements essential to later, more sophisticated tests. Central to all these is a concept hinted at in previous examples of invalid arguments.

Recall that we judged Argument B invalid because it seems possible for the premises of B to be true while the conclusion is false.

Argument B

$$1+1=2$$

$$2+2=4$$

(so,) The first U.S. president was born in Boston.

Such a case reveals the invalidity of an argument because a *valid* argument should be immune to precisely this possibility. With a valid argument true premises are accompanied by true conclusion *without exception*, so there's no *possible* way of having true premises without a true conclusion. If there *is* a possible way for Argument B to have true premises but false conclusion, then Argument B doesn't fit the definition of a "valid argument".

We call such a possible situation a **validity counterexample** (or "counterexample," for short).

A validity counterexample for an argument is a possible situation where the premises of the argument are all *true*, but the conclusion is *false*.

For Argument B, the actual world – where $1+1$ does indeed equal 2, and $2+2$ equals 4, but the first U.S. president was not born in Boston – served as a validity counterexample. But we saw that with some arguments we need to stretch our imaginations to find a validity counterexample. Argument D illustrates this.

Argument D

$$1+1=2$$

$$2+2=4$$

(so,) The first U.S. president was born in Westmoreland County, Virginia.

In the actual world, the premises and conclusion of D are all true. But that isn't enough to make D valid – since it's still *possible* for the argument to have true premises with a false conclusion. With just a bit of imagination we described such a possibility: a situation where John Adams won the first U.S. election, while the mathematical facts remained the same. That situation would qualify as a validity counterexample for Argument D – establishing that the argument is *invalid*.

Just one validity counterexample is sufficient to prove an argument invalid. And for that reason the search for validity counterexamples takes center stage when testing an argument for validity. Roughly speaking: when testing an argument for validity we **try to think of a validity counterexample** for that argument. If we succeed in thinking up such a possibility, we know that the argument is invalid – and if invalid, unconvincing.

We won't rest content with this seat-of-the-pants, imagination-based test of validity. But already it provides an opportunity to rehearse the core concepts of **validity** and **validity counterexamples**, before grappling with more advanced tests. For even in those later tests, those two notions remain central.

2. Validity Counterexample Examples.

The following simple argument supplies a bit of practice in picking out validity counterexamples.

1. The first 43 presidents of the US have been white men.

∴ All presidents of the US (past, present, future) are white men.

We walk through a series of possibilities, asking for each whether it qualifies as a validity counterexample for this argument. First, consider Situation A.

Situation A: The first 43 US presidents are white men; after that they're all Asian women.

In Situation A the premise of our argument would be true. But the conclusion of the argument – that “All presidents of the US (past, present, future) are white men” – is certainly false in A. Since Situation A makes all the premises of the argument true (all *one* of them) while making the conclusion false, Situation A qualifies as a **validity counterexample** for the argument.

Argument	Situation A
1. The first 43 presidents of the US have been white men.	TRUE
∴ All presidents of the US (past, present, future) are white men.	FALSE

Thanks to Situation A, we know this argument is **invalid**: true premises do **not** guarantee a true conclusion here, so the conclusion does **not** follow from the premise.

For a bit more practice, we continue with Possible Situation B.

Situation B: The US ends up having 102 presidents in all, and they're all white men.

In B the premise of the argument is **true**: if all 102 presidents are white men, then certainly the first 43 are. And here the conclusion is **true** as well: all the US presidents ever are white men.

Argument	Situation B
1. The first 43 presidents of the US have been white men. <hr/>	TRUE
∴ All presidents of the US (past, present, future) are white men.	TRUE

Situation B is **not** a validity counterexample for this argument. This situation tells us **nothing** about the validity of the argument.

How about Possible Situation C?

Situation C: Every president, from the first to the last, is a Hispanic man.

In such a situation the premise of the argument would be **false**: here it is not true that the first 43 US presidents are white men. The conclusion would be **false** in C as well.

Argument	Situation C
1. The first 43 presidents of the US have been white men. _____	FALSE
∴ All presidents of the US (past, present, future) are white men.	FALSE

Situation C does **not** qualify as a validity counterexample for this argument. C tells us **nothing** about this argument's validity.

Finally, consider Possible Situation D.

Situation D: The US has 100 white men as presidents, and then a mix of 100 white, black, and Hispanic men, and Eastern European women.

Here the premise of the argument is true: since the first 100 US presidents are white men here, the first 43 are. But it's false in D that all US presidents are white men.

Argument	Situation D
1. The first 43 presidents of the US have been white men. _____	TRUE
∴ All presidents of the US (past, present, future) are white men.	FALSE

Situation D **is** a validity counterexample for our argument. All by itself, Situation D proves that the argument is **invalid**.

We said that Situations B and C tell us nothing about the validity of the argument. Here's why.

Situations A and D already established that this argument is invalid. But Situation B shows us something important about an invalid argument: **an invalid argument can sometimes (by lucky coincidence) have true premises and true conclusion.**

Argument	Situation B
1. The first 43 presidents of the US have been white men. <hr/>	TRUE
∴ All presidents of the US (past, present, future) are white men.	TRUE

Of course a **valid** argument can also have true premises and conclusion – as this example shows.

Valid Argument:

- 1. George Washington was the first US president.**
 - 2. George Washington was born in Virginia**
-

∴ The first US president was born in Virginia.

In the actual world, for instance, the premises and conclusion of this valid argument are true.

Having true premises and true conclusion in a given situation is something both valid *and* invalid arguments can do. So a situation like that – where the argument's premises and conclusion are all true – is no help in settling whether the argument in question is valid or invalid.

In Situation C the above invalid argument had false premises and a false conclusion.

Argument	Situation C
1. The first 43 presidents of the US have been white men. <hr/>	FALSE
∴ All presidents of the US (past, present, future) are white men.	FALSE

But that can happen to a valid argument as well.

Valid Argument:

- 1. Benjamin Franklin was the first US president.**
 - 2. Benjamin Franklin was born in Florida**
-
- ∴ The first US president was born in Florida.**

This argument is valid, because if the premises **were** both true, the conclusion would have to be true as well. Of course, in the actual world both the premises and conclusion are **false**. So: a valid argument can have false premises and conclusion in a certain situation.

Here again, having false premises and false conclusion in a given situation is something both valid *and* invalid arguments can do. So finding such a situation – where an argument’s premises and conclusion are both false – tells us nothing about whether that argument is valid.

As single situations go, the **only** case that tells us anything about an argument’s validity is a situation where the argument has true premises and a false conclusion – a **validity counterexample**. For that’s the one sort of situation a valid argument will never find itself in.

So if, by scouring the world before us or any other possibilities the imagination can dream up, we find such a situation for a given argument, we know for certain that the argument is invalid. That's why the search for validity counterexamples plays such a central role in tests of validity.

Summary: Validity Counterexamples

- A **validity counterexample** for an argument is a possible situation where the argument has true premises but a false conclusion.
- Finding a validity counterexample for an argument establishes that the argument is **invalid**.